HEAT EXCHANGE IN JET COOLING SYSTEMS

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Results are presented from an experimental study of heat exchange in a channel with jet delivery of a coolant. Criterional equations are proposed to calculate the effective temperature of the flow and local heat-transfer coefficients.

Formulation of the Problem and Description of the Experimental Unit. The cooling of surfaces by jets is widely used in power plants. In connection with this, there has recently been considerable study of heat exchange in jet cooling systems. The literature contains fairly detailed recommendations on calculating heat transfer in the flow of a single jet onto a surface in the absence of a longitudinal entraining flow and with the assumption of equality of the temperatures in the jet and in the environment [1-4]. Isolated studies have been made of heat transfer in the flow of a jet on a surface in the presence of a lengthwise flow in a channel [2, 5-7], but most of these works examined the range of small values of the injection parameter m_0, representing the ratio of the initial mass velocity of the jet to the mean mass velocity of the longitudinal flow ($m_0 = (\rho u)_{s0}/(\rho u)_{f2} \leqslant 12$). The correlations suggested in [6] were obtained for mean heat-transfer coefficients on a specific model with equality of the temperatures in the jet and the longitudinal flow. In actual jet systems there is often interaction between individual jets and a nonisothermal longitudinal flow (either autonomously or by the formation of neighboring jets after the initial jets reach the surface), at both small and large values of the injection parameter up to $m_0 \to \infty$.

The present work studies the process of heat exchange in an annular channel. A gas jet is delivered through one wall of the channel (Fig. 1a). To develop a method of calculating heat transfer under these conditions, we have to study the laws of the distribution of the local heat-transfer coefficients in the flow of a jet on a surface when the longitudinal flow in the channel is nonisothermal. We propose to take a new, semiempirical approach to analyzing the heat-exchange process in the channel on the basis of superposition of the solutions for each jet in the system individually.

Features of the heat-exchange process being studied include:

1) occurrence of the heat exchange with a variable gas discharge along the channel due to mixing of the jet (transverse) flow in the longitudinal gas flow, the jets of the jet flow here intensifying the heat-exchange process;

2) formation of a boundary layer (film) on the inside surface of the outer wall as a result of flowing of the jets over the channel surface;

3) occurrence of heat and mass exchange between the individual jets of the transverse flow and the longitudinal gas flow.

Given the above conditions, the temperature of the heat carrier will be quite nonuniform both over the height and over the surface of the channel. Thus, there arises the problem of choosing the determining temperature of the flow when calculating the heat-transfer coefficient in accordance with Newton's formula. It was suggested that the effective temperature of the flow [8] be used as the determining temperature, which makes it possible to allow for the change in the temperature of each jet in its interaction with the longitudinal flow under the condition that their temperatures differ. By the effective temperature, we mean the temperature that the thermally insulated surface of the channel would have from the interaction of the transverse jet flow with the nonisothermal lengthwise flow. Then we can write Newton's formula for the heat-transfer coefficient in the form

$$\alpha = \frac{q_w}{T_w - T_e}, \tag{1}$$

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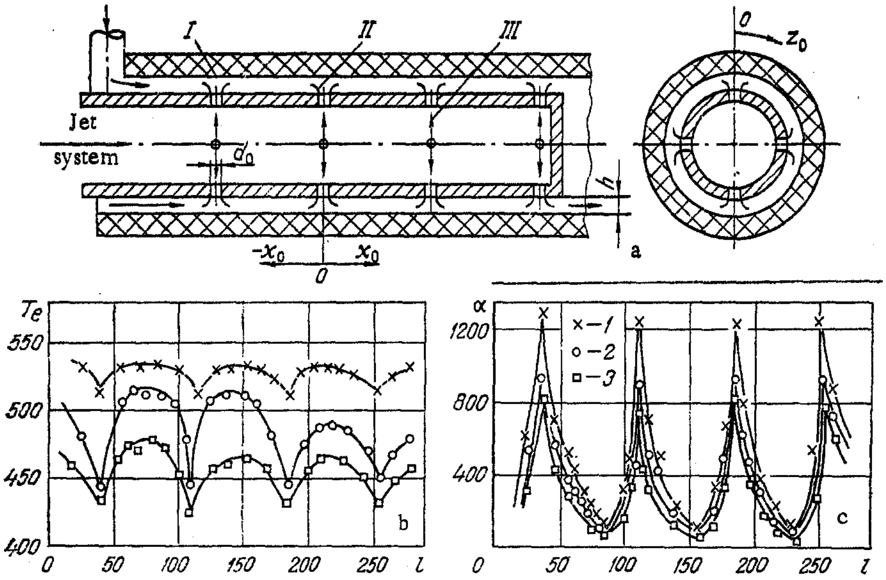

Fig. 1. Distribution of the effective temperature of the flow (b) and the heat-transfer coefficients (a) for a system of four rows of holes: 1) $G_{f_{10}}/G_{f_{20}}$ = 0.55; 2) 1.07; 3) 0.31. (I — test surface; II — jet; III — transverse (jet) flow).

where q_w is the heat flux on the channel surface; T_w is the temperature of the surface.

The heat-exchange process was studied experimentally in an annular channel (Fig. 1a). Air was blown from a system of holes through the perforated inside wall (transverse jet flow). An independent flow of air was also provided along the channel (longitudinal flow). The experimental unit allowed us to preheat the longitudinal flow and to heat the outer wall of the annular channel by the passage of a low-voltage alternating current. The working substance was air. The outer surface of the test section was protected with a heat-insulating shell to prevent heat loss. The temperature of the surface was measured with 27 thermocouples. The test section was designed so as to permit rotation of the inside tube to measure the temperature field about the perimeter of the channel. The height of the channel was regulated by changing the diameter of the inside tube.

The tests were conducted in the following ranges of the basic parameters: injection parameter $m_0 = (\rho u)_{s0}/(\rho u)_{f2} \geqslant 6$; Reynolds number in the longitudinal flow $Re_{f_2} = (\rho u)_{f_2} d_{eq}/\mu = 3 \cdot 10^3 - 3 \cdot 10^4$; degree of nonisothermality of the longitudinal flow (ratio of the initial temperatures in the longitudinal flow and in the jet) $T_{f_{20}}/T_{s_0}$ = 1-2; heat flux on the surface $q_w = 6 \cdot 10^3 - 1.4 \cdot 10^5$ W/m^2; relative height of annular channel $H = h/d_0$ = 3-8.5; relative spacing of the jets $\Delta\bar{x} = \Delta x/d_0 \geqslant 13$.

The heat-transfer study was conducted in two stages.

1. We studied laws of change in the effective temperature over the channel surface. For this purpose, we thermally insulated the outer wall of the channel and preheated the longitudinal gas flow. Also, we measured the temperature distribution over the channel surface in the tests.

2. We established laws of change in the local heat-transfer coefficients over the channel surface. Here, the outer wall was heated by the passage of an electrical current through it, and we determined the local heat-transfer coefficient.

The analysis was performed for the neighborhood of the critical point of jet flow and in the region of the boundary flow. The analysis was based on laws of the discharge of a jet into an infinite longitudinal flow. The effect of the longitudinal flow on the basic characteristics of the jet can be determined in accordance with experimental studies [9-11]. Test data on the change in axial velocity u_{sm} along the path of the jet was generalized by an equation permitting an asymptotic solution to the case of a submerged jet ($m_0 \to \infty$):

when H 6.2

$$\frac{u_{sm} - u_{f2}\cos\beta}{u_{s0}} = 1 - \exp\left(-0.5\frac{m_0^{1.8}\sqrt{\rho_{f2}/\rho_s}}{\bar{s}^2}\right), \qquad (2)$$

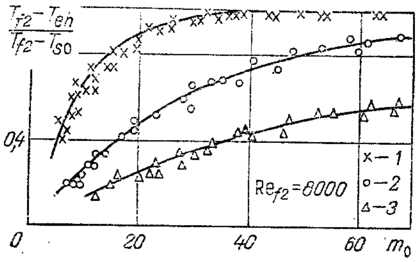

Fig. 2

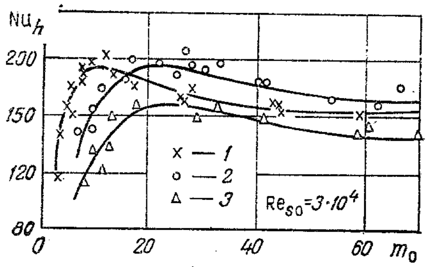

Fig. 3

Fig. 2. Dependence of relative effective temperature of flow in the vicinity of the critical point of jet flow on the injection parameter: 1) H = 3.5; 2) 6.5; 3) 8.5.

Fig. 3. Dependence of the heat-transfer coefficient in the vicinity of the critical point of the jet flow on the injection parameter. Notation is the same as in Fig. 2.

when H > 6.2

$$\frac{u_{sm}-u_{f2}\cos\beta}{u_{s0}}=\left[1-\exp\left(-0.039\frac{m_0^{1.8}\sqrt{\rho_{f2}/\rho_s}}{\bar{s}^{0.6}}\right)\right]\frac{6.2}{\bar{s}}, \tag{3}$$

where β is the angle of inclination of the jet path to the longitudinal flow; $\bar{s} = s/d_0$.

The length of the jet path s and the angle of its inclination β can be determined on the basis of the correlation for the jet axis [11]

$$\frac{y}{d_0}=0.73m_0^{1.04}\left(\frac{\rho_{f2}}{\rho_s}\right)^{0.41}\left(\frac{x}{d_0}\right)^{0.29}, \tag{4}$$

assuming y = h.

Results of Study of the Effective Flow Temperature. Experimental determination of the effective temperature reduced to measuring the temperature of the thermally insulated surface. The completed tests showed (Fig. 1b) that the temperature field of the channel surface is quite nonuniform.

To analyze the laws of change in the relative effective temperature $\Theta_e=(T_{f2}-T_e)/(T_{f2}-T_{s0})$, we have to know the distribution of the mean-mass temperature in the annular channel T_{f2} and the temperature of the gas along the inside tube T_{f1}, which determines the initial temperature of the gas in the jet T_{s0}. The temperature of the gas in the inside tube, with allowance for heat transfer through its wall, was determined experimentally with a movable antenna-type thermocouple. It was found that the law of change in the temperature along the inside tube for the case of a thermally insulated channel surface is satisfactorily described by the relation

$$\Theta_{f1}=\frac{T_{f1}-T_{f10}}{T_{f1l}-T_{f10}}=\frac{\bar{x}}{10-9\bar{x}}, \tag{5}$$

where $\bar{x} = x/l$ is the relative coordinate along the tube axis.

Experimental studies [2, 6, 7] have shown that, with a sufficiently large spacing between jets $(\Delta\bar{x}\geqslant 13)$, the distribution of gas discharge in the jet system is nearly uniform. Then the mean-mass temperature of the gas in the annular channel can be determined by solving the heat balance equation for an annular channel and an internal tube, which is written as follows in this case

$$(G_{f20}+G_{f10}\bar{x})\,c_p\frac{dT_{f2}}{dx}+(G_{f10}-G_{f10}\bar{x})\,c_p\frac{dT_{f1}}{dx}+\frac{G_{f10}}{l}c_p(T_{f2}-T_{f1})=0, \tag{6}$$

where G_{f10} and G_{f20} are respectively the initial air discharge into the inside tube and the annular channel.

Using the solution of Eq. (6) with allowance for (5), we find

$$\Theta_{f2}=\frac{T_{f2}-T_{f20}}{T_{f2l}-T_{f20}}=\frac{\bar{x}\left(\frac{G_{f20}}{G_{f10}}+1\right)}{\frac{G_{f20}}{G_{f10}}+\bar{x}}\left[1+\frac{(T_{f1l}-T_{f10})(\bar{x}-1)}{(10-9\bar{x})(T_{f10}-T_{f20})}\right]. \tag{7}$$

Figure 2 shows results of our study of the effective temperature of the flow in the neighborhood of the critical point of the jet flow. The experimental data is satisfactorily generalized by a criterional relation of the form

$$\Theta_{eh} = \frac{T_{f2} - T_{eh}}{T_{f2} - T_{s0}} = 1 - \exp(-0.06 m_0^{1,1} H^{-2} \mathrm{Re}_{f2}^{0,32}) \text{ when } H \leqslant 6.2, \tag{8}$$

$$\Theta_{eh} = \frac{T_{f2} - T_{eh}}{T_{f2} - T_{s0}} = 1 - \exp(-0.06 m_0^{1,1} H^{-2} \mathrm{Re}_{f2}^{0,32}) \frac{6.2}{H} \text{ when } H > 6.2.$$

Here, the Reynolds number in the longitudinal flow (Re_{f2}) was determined from the total discharge in the given section of the channel with allowance for mixing of the jet flow into the longitudinal flow. As the determining temperature, we used the mean-mass temperature of the gas in the given section of the annular channel, while we used the equivalent diameter of the channel $d_{eq} = 2h$ as the characteristic dimension.

In finding criterional relations to calculate the effective temperature of the flow after the jet reaches the surface, as scales it is best to choose the temperature in the vicinity of the critical point and the velocity of flow of the jet on the surface. By the jet flow velocity on the surface, we mean the velocity that would exist on its axis in the case of a free jet at a distance from the initial section equal to the height of the channel ($y = h$). The velocity of the jet on its axis in flowing over the surface (u_{sh}) is calculated with allowance for the interaction of the jet with the longitudinal (entraining) flow by means of Eqs. (2) and (3).

Use of the injection parameter, determined from the jet flow velocity $m_h = (\rho u)_{sh}/(\rho u)_{f2}$, as the determining criterion allows us to generalize the test results from the study of the effective flow temperature with a single criterional relation for annular channels of different heights:

Along the x axis

$$\Theta_e(x_0) = \frac{T_{f2} - T_e(x_0)}{(T_{f2} - T_e)_h} = \exp[-A\bar{x}_0 f(m_h)], \tag{9}$$

along the z axis

$$\Theta_e(z_0) = \frac{T_{f2} - T_e(z_0)}{(T_{f2} - T_e)_h} = \exp[-A\bar{z}_0 \varphi(m_h)].$$

Here, $A = 0.112$; $f(m_h) = 1 - m_h^{-0.53}$ in the direction of the longitudinal flow; $f(m_h) = 1 + m_h^{-0.65}$ in the direction opposite the longitudinal flow; $\varphi(m_h) = 1 + m_h^{-1.1}$; the coordinates $\bar{x}_0 = x_0/d_0$ and $\bar{z}_0 = z_0/d_0$ were reckoned from the critical point of the jet flow on the surface. The distribution of the mean-mass temperature along the annular channel was determined in accordance with (7).

Criterional relations (8), (9) for the effective temperature T_e are universal in character, since the mean-mass temperature of the flow is used as the scale. In the presence of heat exchange on the surface, the problem of calculating the effective temperature by Eqs. (8) and (9) reduces to determining the mean-mass temperature T_{f2}, which can be found from the heat-balance equation corresponding to the conditions of the specific problem.

Results of Study of Local Heat-Transfer Coefficient. The heat-transfer coefficients were calculated in accordance with Eq. (1). The distribution of the effective temperature of the flow was determined on the basis of the condition that criterional relations (8) and (9), obtained with a thermally insulated channel surface, are also valid for the case of an uninsulated channel surface (in the presence of heat exchange). The effect of heat exchange on the effective flow temperature is manifest in a change in the mean-mass temperature of the flow in the annular channel during the heat exchange.

The change in the mean-mass temperature of the gas along the annular channel can be obtained from the solution of the heat-balance equation

$$(G_{f20} + G_{f10}\bar{x}) c_p \frac{dT_{f2}}{dx} + (G_{f10} - G_{f10}\bar{x}) c_p \frac{dT_{f1}}{dx} + \frac{G_{f10}}{l} c_p (T_{f2} - T_{f1}) = \frac{Q}{l}. \tag{10}$$

The law of change in the gas temperature along the inside tube (as in the case of a thermally insulated surface) was determined experimentally and is satisfactorily generalized by the following relation:

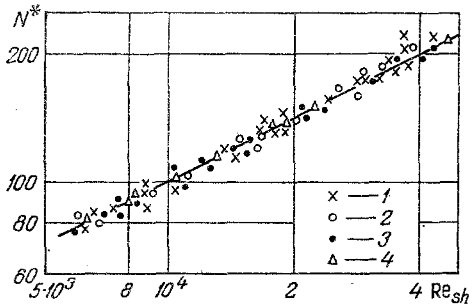

Fig. 4. Heat exchange near the critical point of the jet flow: 1) H = 3.5; 2) 4.25; 3) 6.5; 4) 8.5; $N^* = \{Nu_h\}\{0.0025\sqrt{Re_{s0}}[1 + \exp(-0.64 m_0/m_{opt})] + 0.49\}^{-1}$.

$$\Theta_{f1} = \frac{T_{f1} - T_{f10}}{T_{f1l} - T_{f10}} = \frac{\bar{x}}{10 - 9\bar{x}^2}. \tag{11}$$

In accordance with the solution of Eq. (10) and with allowance for (11), the expression for the mean-mass temperature of the gas in the annular channel is written thusly

$$\Theta_{f2} = \frac{T_{f2} - T_{f20}}{T_{f2l} - T_{f20}} = \frac{\bar{x}\left(\dfrac{G_{f20}}{G_{f10}} + 1\right)}{\dfrac{G_{f20}}{G_{f10}} + \bar{x}}\left[1 + \frac{(T_{f1l} - T_{f10})(\bar{x} - 1)}{10 - 9\bar{x}^2}\,\frac{G_{f10}c_p}{Q}\right]. \tag{12}$$

Knowing the mean-mass temperature of the gas along the annular channel and the inside tube (the initial temperature of the gas in the jet), we can find the distribution of the effective flow temperature over the surface of the channel in the presence of heat exchange in accordance with criterional relations (8), (9).

Figure 1c shows a typical distribution of the local heat-transfer coefficients over the channel surface. It can be seen from the curves that the maximum value of the coefficient is seen near the critical point of jet flow on the surface. The coefficient decreases with increasing distance from the critical point. These results agree qualitatively with the results in [5].

Heat exchange was analyzed in the vicinity of the critical point and in the region of jet flow over the channel surface. According to studies of heat transfer in the flow of a jet over a surface in the absence of longitudinal flow, heat transfer near the critical point is a function of the Reynolds number of the jet — determined from the initial parameters in the jet ($Re_{s0} = (\rho u)_{s0} d_0/\mu$) — and of the relative height of the channel H.

The completed tests showed (Fig. 3) that in the presence of a longitudinal flow, heat transfer near the critical point depends on the injection parameter m_0 as well as on other factors. Meanwhile, for each value of channel height there is an optimum value of the injection parameter m_{opt} at which the heat-transfer rate is maximal (Fig. 3).

Analysis of the results of experimental study of heat transfer showed that the optimum value of the injection parameter occurs in a region in which, as the parameter increases, the longitudinal flow ceases to affect the change in velocity on the jet axis at the distance h from its initial section. Assuming that the heat-transfer maximum is realized under the condition $u_{sh}/u_{s.h} = 0.95$ (where $u_{s.h}$ is the velocity on the axis of a submerged jet at the distance h, i.e., as $m_0 \to \infty$), from Eqs. (2) and (3) we obtain a relation for determining the optimum value of the injection parameter m_{opt}:

$$m_{opr} = 2.73 H^{1.12}\left(\frac{\rho_{f2}}{\rho_s}\right)^{-0.28} \text{ when } H \leqslant 6.2,$$
$$m_{opr} = 11{,}37 H^{0.34}\left(\frac{\rho_{f2}}{\rho_s}\right)^{-0.28} \text{ when } H > 6.2. \tag{13}$$

The Reynolds number $Re_{sh} = (\rho u)_{sh} d_0/\mu$ is determined from the jet flow rate, which is calculated from Eqs. (2) and (3). The results of the experimental study of the heat-transfer coefficient are satisfactorily generalized by the following criterional relation (Fig. 4):

$$\mathrm{Nu}_h = \left\{0.0025 \sqrt{\mathrm{Re}_{s0}} \left[1 + \exp\left(-0.64 \frac{m_0}{m_{\mathrm{opt}}}\right)\right] + 0.49\right\} \sqrt{\mathrm{Re}_{sh}}. \quad (14)$$

To find the law of the distribution of the local heat-transfer coefficients over the channel surface, we used the heat-transfer coefficient and the velocity at the critical point of jet flow as the scale. The following criterional relations were obtained. They adequately generalize the test results for the distribution of local heat-transfer coefficients over the channel surface

along the x axis

$$\frac{\alpha(x_0) - \alpha_0}{(\alpha - \alpha_0)_h} = \exp[-A\bar{x}_0 f(m_h)], \quad (15)$$

along the z axis

$$\frac{\alpha(z_0) - \alpha_0}{(\alpha - \alpha_0)_h} = \exp[-A\bar{z}_0 \varphi(m_h)],$$

where A = 0.244; $f(m_h) = 1 - m_h^{-0.53}$ in the direction of the longitudinal flow; $f(m_h) = 1 + m_h^{-0.65}$ counter to this flow; $\varphi(m_h) = 1 + m_h^{-1.1}$; α_0 is the heat-transfer coefficient in the channel section of interest with a stabilized, uniform flow of the coolant.

NOTATION

s, coordinate along the jet axis; x, z, coordinates along the generatrix and about the circumference of the annular channel; d, diameter; P, perimeter; h, height of annular channel; l, length of annular channel; G, discharge of coolant; Q, heat flux on outer channel; α, heat-transfer coefficient; u, velocity; T, temperature; T_e, effective flow temperature; $m = (\rho u)_s/(\rho u)_{f_2}$, injection parameter; $Re = \rho u d/\mu$, Reynolds number; $Nu = \alpha d/\lambda$, Nusselt number; $K = \alpha/\alpha_0$, heat-transfer intensification coefficient; μ, absolute viscosity of air; λ, thermal conductivity of air; c_p, specific heat of air; ρ, density of air. Indices: 0, parameters in the initial section of the jet, the annular channel, and the inside tube; l, parameters at the outlet of the annular channel and inside tube; s, parameters of the jet; h, parameters near the critical point of jet flow on the surface; f_2, parameters of the air flow in the annular channel; f_1, parameters of the air flow in the inside tube; w, parameters of the channel wall.

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HEAT TRANSFER OF A CYLINDRICAL FILM-TYPE TRANSDUCER IN A HOT-WIRE ANEMOMETER WITH INTERNAL HEATING

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The article examines the heat transfer of a cylindrical film-type transducer in a hot-wire anemometer with an internal heat source operating in a moving medium.

One of the more promising directions being taken in the development of thermoanemometric measurement technology, employed in experimental aerohydrodynamics, is the use of transducers in the form of a dielectric base with a superimposed metallic film [1, 2]. Theoretical and experimental studies show that the process of heat exchange between the film and the dielectric base has a significant effect on the metrological characteristics of the transducer [3]. Heat exchange between the film and base can be reduced by using an internal heat source to create a temperature gradient between them. To analyze the operation of such a transducer, we will examine a model in the form of a finite cylinder of length $2l$ with a surface heat source of length 2n. An internal heat source ω is located in the axis of the cylinder (Fig. 1). Since the local heat-transfer coefficient of the heated cylinder, immersed in the flow of the medium, depends on the angle φ, then its heat-conduction equation has the form [4]

$$a\left[\frac{\partial^2}{\partial r^2}T(r,\ z,\ \varphi)+\frac{1}{r}\frac{\partial}{\partial r}T(r,\ z,\ \varphi)+\frac{\partial^2}{\partial z^2}T(r,\ z,\ \varphi)+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}T(r,\ z,\ \varphi)\right]+\frac{\omega}{c\gamma}=0. \tag{1}$$

However, the temperature field of the substrate under the surface heat source is nearly axisymmetrical, and Eq. (1) takes the form

$$a\left[\frac{\partial^2}{\partial r^2}T(r,\ z)+\frac{1}{r}\frac{\partial}{\partial r}T(r,\ z)+\frac{\partial^2}{\partial z^2}T(r,\ z)\right]+\frac{\omega}{c\gamma}=0. \tag{2}$$

The boundary conditions

$$-\lambda\frac{\partial}{\partial r}T(R,\ z)+g-\alpha[T(R,\ z)-T_{\mathrm{md}}]=0, \tag{3}$$

$$T(0,\ z)\neq\infty, \tag{4}$$

$$\frac{\partial}{\partial z}T(r,\ 0)=0, \tag{5}$$

$$\frac{\partial}{\partial z}T(r,\ n)=0. \tag{6}$$

Using (5) and (6), we execute a finite Fourier cosine transform with respect to the coordinate z [5]:

$$r\frac{d^2}{dr^2}T(r,\ p)+\frac{d}{dr}T(r,\ p)-rp^2T(r,\ p)+r\frac{\omega l\sin\frac{p\pi c}{l}}{c\gamma ap\pi}=0 \tag{7}$$

and set the boundary conditions:

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